arXiv:gr-qc/0203079v1 22 Mar 2002

On the pre-nucleosynthesis cosmological period

R. Aldrovandi

Instituto de Física Teórica
State University of São Paulo - UNESP
Rua Pamplona 145
01405 - 900 São Paulo Brazil

J. Gariel and G. Marcilhacy

Laboratoire de Gravitation et Cosmologie Relativistes
Université Pierre et Marie Curie, CNRS/ESA 7065
4, Place Jussieu Cedex 05
75252 Paris, France

Abstract

Physics, as known from our local, around–earth experience, meets some of its applicability limits at the time just preceding the period of primeval nucleosynthesis. Attention is focussed here on the effects of the nucleon size. Radiation–belonging nucleons are found to produce an extremely high pressure at $kT \approx$ some tens or hundreds of MeV. Quark deconfinement at higher energies would not change the results.

1 Introduction

The standard procedure of Physical Cosmology is to take present-day knowledge and data and travel backwards in time, applying as well as possible our local, laboratory- and observatory-tested Physics. That Physics, as we know it today, is able to explain so many of the progressively distant and red–shifted data is the best mark we have of its astounding range of validity. The remotest time for which we have reliable results is the nucleosynthesis epoch: well–established Physics is able to give a fair account of the cosmological origin of the lightest elements. We shall here be interested in the period just preceding that nucleosynthesis era, which we shall call “pre–nucleosynthesis period” (PNS period).

The end of that period – the beginning of the nucleosynthesis age – must correspond to a temperature kT of the order of the deuteron binding-energy,

which means a few MeVs and a red-shift $z \approx 2 \times 10^{10}$. The PNS period could also be called the "close-packing period". The "closely-packed" constituents referred to are protons or, more precisely, nucleons. A rough estimate gives for their concentration a value around $10^{23} cm^{-3}$ and for their mean free path $\lambda \approx (n_b \sigma)^{-1} \approx (10^{23} \times\ 10^{-26})^{-1} = 1000\ cm$. The usual approximation assuming ideal fluids fails, but the physical assumptions on the large ratio between the total volume of the system and the total volume occupied by the constituents are valid and current Physics can be expected to hold.

Protons are remarkably stable (lifetime larger than 1.6×10^{25} years [1]), and neutrons decay into protons. Thus, we can safely suppose that the nucleons present today in the Universe have been around from the "beginning". The values of the critical density and baryon density [see below, equation (12)] imply a remnant nucleon density n_N in the range $0.059 \leq n_N \leq 0.296$ $(nucleon \times m^{-3})$ at present time. Now, each nucleon occupies a volume of the order $2.2 \times 10^{-45} m^3$, which means that at $z \approx 10^{15}$ they attain a tightly packed state: one nucleon per nucleon volume. This will define for us the beginning of the PNS period. The mean free path is then of the order of the size of the constituents. In a nutshell: the causally-related Universe has a volume $V_U \approx 10^{81} cm^3$ and contains $N_N \approx 10^{74}$ nucleons. The total "internal" volume of these nucleons is $V_n \approx 10^{35} cm^3$. The Universe had that volume when $z \approx 10^{15}$. The assumption of infinite system volume – which underlies the thermodynamic limit, as well as the very definition of cross-section – is then at least doubtful, and the ideal fluid hypothesis is clearly untenable. We shall later refine this crude estimate, but the result will be, not quite surprisingly, essentially the same for the remnant protons. The interest of the more refined approach rests on its formulas, which can be applied to the protons belonging to the radiation bath.

The PNS period runs consequently between $z \approx 10^{10}$ and $z \approx 10^{15}$. The strategy to be followed will be rather circular. Quark deconfinement will be ignored to start with and protons will be taken as stable. We shall describe them by a potential and find that the pressure related to present–day matter tends to an infinite value. That would happen, however, at energies for which the notion of potential does not apply and for which deconfinement is quite possible. We then reconsider the question from the point of view of the radiation–belonging nucleons, and find the same effect at much lower energies, for which potentials do have a meaning and there is no possibility of deconfinement.

In a first dealing with such unusual conditions we shall feel justified in

taking a naïve approach. Instead of facing the intricacies of the high-density matter equation of state [2], we shall content ourselves with reasonable order–of–magnitude estimates. The proton will be considered as a hard constituent, occupying an irreducible hard-core volume $\simeq 1 fermi^3 \simeq 10^{-39} cm^3$, represented by a hard-sphere potential. Potentials have been used from time to time in Cosmology, for instance to show how the initial singularity can be avoided [3]. They should, of course, be carefully handled in relativistic conditions. We shall be attentive to the energy conditions under which the very notion of potential can loose its validity.

Section 2 is a summary of the Standard Model, actually a commented formulary devoted to fixing notation, showing the numbers we use and summing up some observation values relevant to our subject. We shall ignore non-standard possibilities, as eventual "dark" constituents, and accept usual reasonable assumptions, such as the Debye screening which renders electrostatic effects negligible. Such a review of well–known topics is necessary to show how and when we part from the standard procedure. In section 3 a general overview of physical problems appearing in the PNS period is given. In particular, we present our assumption that protons keep their identity in energies much higher than usually supposed. We then proceed to a discussion of the hard-sphere potential and to a naïve application to the Friedmann equations. The result is that an infinite matter pressure would block the backward progress at around $z \approx 10^{15}$ if we take into account only the "remnant" protons existing at present time. At those red–shifts the nucleons are relativistic and the idea of a hard-sphere potential is unrealistic, but it is easier to argue starting from the consideration of the remnant present–day protons. We show then (section 5) that the protons appearing through pair-creation from the radiation background produce the same effect at much lower energies. Pairs of photons with energy barely enough to produce proton–antiproton pairs will create non-relativistic protons and anti–protons, and for these the notion of a potential does make sense. Pair creation is a very efficient process: the number of created protons is very large already at energies much lower than 1 GeV. The pressure blockage, in consequence, takes place at rather low red–shifts. The possible meanings of these results are discussed in the last section. A briefing on relativistic quantum gases is given in Appendix A.

2 The standard model

The large scale evolution of the Universe is described [4, 5] by the two Friedmann equations for the scale parameter $a(t)$:

$$\dot{a}^2 = \left[2\left(\frac{4\pi G}{3}\right)\rho + \frac{\Lambda c^2}{3}\right] a^2 - kc^2 \tag{1}$$

$$\ddot{a} = \left[\frac{\Lambda c^2}{3} - \frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)\right] a(t) \tag{2}$$

which, once combined, lead to the two equivalent expressions

$$\frac{d\rho}{dt} = -3\ \frac{\dot{a}}{a}\ \left(\rho + \frac{p}{c^2}\right)\ , \tag{3}$$

$$\frac{d}{da}(\epsilon a^3) + 3\ p\ a^2 = 0. \tag{4}$$

This equation can be alternatively obtained from the vanishing of the covariant divergence of the source energy-momentum tensor, and reflects simply energy conservation. Notation is hopefully obvious: $\rho = \epsilon/c^2$ is the source energy density in mass, p the pressure, Λ the cosmological constant. The index "0" will indicate present–day values: the red-shift z, for example, is given by

$$1 + z = \frac{a(t_0)}{a(t)}\ . \tag{5}$$

The Hubble function

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{d}{dt}\ln a(t) \tag{6}$$

has the present-day value $H_0 = 100\ h\ km\ sec^{-1}\ Mpc^{-1} = 3.24{\times}10^{-18}\ h\ sec^{-1}$ (with the parameter h, of the order of unity, encapsulating the uncertainty in present-day measurements).

Given an equation of state in the form $p = p(\rho)$, equation (4) can be integrated to give

$$1 + z = \frac{a_0}{a(t)} = e^{\frac{1}{3}\int_{\epsilon_0}^{\epsilon}\frac{d\epsilon}{\epsilon + p(\epsilon)}}\ . \tag{7}$$

For example, for a pure radiation content the equation of state is $p = \frac{1}{3}\epsilon$, so that

$$\epsilon_z = \epsilon_0\ (1+z)^4\ .$$

The energy density of dust matter, with $p = 0$, will behave according to

$$\epsilon_z = \epsilon_0 \; (1+z)^3 \; .$$

Recall that Eq.(4) is a mere consequence of energy conservation. These results are independent of the parameters k and Λ. The relationship between z and the Hubble function is easily found. Taking the time derivative of Eq.(5) and comparing with the last expression, one arrives at $\frac{dz}{dt} = -\, H(t)(1+z)$, which integrates to

$$1 + z = e^{- \; \int_{t_0}^{t} H(t)dt} \; . \tag{8}$$

The critical mass density is

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-26} \; h^2 \; kg \times m^{-3} \; . \tag{9}$$

The baryon concentration and mass density are given by

$$n_b = 11.2 \; \Omega_{b0} \; h^2 \; (1+z)^3 \;\; m^{-3} \tag{10}$$

$$\rho_b = 1.88 \times 10^{-26} \Omega_{b0} \; h^2 \; (1+z)^3 kg \; m^{-3}, \tag{11}$$

where the parameter $\Omega_{b0} = \frac{8\pi G}{3} \frac{\rho_{b0}}{H_0^2} = \frac{\rho_{b0}}{\rho_{crit}}$ has observational values in the range

$$0.0052 \; \leq \; \Omega_{b0} h^2 \; \leq 0.026 \; . \tag{12}$$

These values for the critical and baryon density lead to the remnant nucleon density range used in the Introduction. In terms of $H(t)$, the equations can be written as

$$H^2 = 2 \left(\frac{4\pi G}{3} \right) \rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{13}$$

$$\dot{H} = - \; 4\pi \; G \left(\rho + \frac{p}{c^2} \right) + \frac{kc^2}{a^2} \tag{14}$$

$$\dot{a}(t) = H(t) \; a(t) \; ; \; \frac{d\rho}{dt} = -3H(\rho + \frac{p}{c^2}) \; . \tag{15}$$

Introducing $\Omega_m = \frac{\rho}{\rho_{crit}}$, $\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$ and $\Omega_k(t) = - \; \frac{kc^2}{a^2 H_0^2}$, equation (13) takes the form

$$\frac{H^2}{H_0^2} = \frac{\rho}{\rho_{crit}} - \frac{kc^2}{a^2 H_0^2} + \frac{\Lambda c^2}{3H_0^2} = \Omega_m + \Omega_k + \Omega_\Lambda \; . \tag{16}$$

Notice that Ω_m refers to the total amount of content: if baryons and radiation are to be considered, then $\Omega_m = \Omega_b + \Omega_\gamma$. The above expression gives on present-day values the constraint

$$\Omega_{m0} + \Omega_{k0} + \Omega_\Lambda = \Omega_{b0} + \Omega_{\gamma 0} + \Omega_{k0} + \Omega_\Lambda = 1. \tag{17}$$

We have used $\Omega_{k0} = -\frac{kc^2}{a_0^2 H_0^2}$, with $a_0 = a(t_0)$. There is a recent evidence for a large value of Ω_Λ and a small value of Ω_{k0} [6]. Choosing for time and length the convenient units

$$H_0^{-1} = 3.0857 \times 10^{17} h^{-1}\ sec\ ;\ \frac{c}{H_0} = 9.25 \times 10^{25} h^{-1}\ \ m\ , \tag{18}$$

the Friedmann equations acquire the simpler forms

$$H^2 = \frac{\rho}{\rho_{crit}} - \frac{k}{a(t)^2} + \frac{\Lambda}{3} \tag{19}$$

$$\dot{H} = -\ \frac{3}{2}\ H^2 -\ \frac{3}{2}\ \frac{p}{c^2 \rho_{crit}}\ +\ \frac{\Lambda}{2} -\ \frac{1}{2}\ \frac{k}{a(t)^2}\ . \tag{20}$$

Gases at high energies in the presence of pair production are also better considered in adapted units, which we introduce here while leaving details to Appendix A. First of all, given a particle of mass m, it is convenient to use

$$\tau = \frac{kT}{mc^2} \tag{21}$$

as the temperature variable. There are also two lengths of major interest, the Compton length and the thermal wavelength. The static Compton length is a most natural unit of length:

$$\lambda_C = \frac{\hbar c}{mc^2}\ . \tag{22}$$

For the electron and for the proton, respectively, $\lambda_e = 3.81 \times 10^{-11}\ cm$ and $\lambda_p = 2.08 \times 10^{-14}\ cm$. A natural volume cell for the proton will be $\lambda_p^3 = 9.0 \times 10^{-42}\ cm^3$.

If $\beta = 1/kT$ is the inverse temperature, (the cube of) the relativistic generalization [7] of the thermal wavelength is given by

$$\Lambda_T^3(\beta) = 2\ \pi^2 \beta mc^2\ \frac{e^{-\beta mc^2}}{K_2(\beta mc^2)}\ \left(\frac{\hbar c}{mc^2}\right)^3 = \frac{2\ \pi^2}{\tau}\ \frac{e^{-1/\tau}}{K_2(1/\tau)}\ \lambda_C^3\ . \tag{23}$$

Here $K_2(x)$ is the modified Bessel function of second order, whose asymptotic behaviors, leading to the nonrelativistic and the ultra–relativistic limits, are given in Appendix A. Here we only remark that the non-relativistic limit gives the usual expression

$$\Lambda_{NR}(\beta) = \lambda = \hbar \sqrt{\frac{2\pi\beta}{m}} = \sqrt{\frac{2\pi}{\tau}}\ \lambda_C\ . \tag{24}$$

For instance, a proton at $kT \approx 4\ MeV$ will have $\Lambda_{NR} \approx 40\ \lambda_p$. A proton will "occupy" a degeneracy-volume $\lambda^3 = \frac{1.42\times 10^{-40}}{\tau^{3/2}}\ cm^3$, from which every other proton will be statistically excluded by Fermi repulsion. The ultra-relativistic limit will be

$$\Lambda_{UR}(\beta) = \pi^{2/3}\ \beta\hbar c = \pi^{2/3}\ \frac{\lambda_C}{\tau}\ . \tag{25}$$

The pressure p and the mass density ρ in (1) and (2) are those of matter and radiation present in the Universe, introduced through their expressions for ideal gases. Interactions are only taken into account through reactions supposed to take place in restricted conditions. As examples, a weak Thomson scattering lies behind thermal equilibrium before recombination, and pair production will be responsible for the existence of a huge number of electrons when kT is higher than $\approx 0.5 MeV$.

The values $\Lambda = 0$, $k = 0$ lead to very simple solutions and are helpful in providing a qualitative idea of the general picture. They will be used as reference cases. We shall later exhibit the expression of $H(z)$ in the pre–recombination period, as well as the implicit expression of $a(t)$. Nevertheless, in order to get a firmer grip on the relevant contributions and the role of each term, it is useful to review the customary discussion on the matter- and radiation- dominated ages.

2.1 Matter–dominated age

Always in the standard approach, baryons (essentially nucleons) dominate the energy content at present time. This domination goes back to the "turning point" time given below (equation (48)), when radiation takes over. Protons are non-relativistic during all this period. The standard argument runs as follows. Matter pressure has the expression $p_b = n_b kT_b$. It appears, however, always in the combination $\rho_b + p_b/c^2 = n_b[m + kT_b/c^2] = \frac{n_b}{c^2}[mc^2 + kT_b]$. Thus, p_b is negligible in the non-relativistic regime. Putting $p = 0$, the

reference case with $\Lambda = 0$ and $k = 0$ has for equations

$$\dot{H} = -\,\frac{3}{2}\,H^2 = -\,\frac{3}{2}\frac{\rho_b}{\rho_{crit}}\ .$$

The general solution is

$$\frac{1}{H\,(t)} = \frac{3}{2}\ t + C. \tag{26}$$

Let us quickly examine 3 models, two with $k = 0$, $\Lambda = 0$ and a third case with $\Lambda \neq 0$.

a) Matter–domination: dust Universe

This unrealistic model supposes matter domination all the time along. It takes at the "beginning" $H_{t=0} = \infty$. The integration constant C vanishes and the solution is simply

$$H(t) = \frac{2}{3t}\ .$$

This means that

$$\frac{da}{a} = \frac{2}{3}\ \frac{dt}{t}\quad .$$

The expressions relating the Hubble function, the expansion parameter, the red-shift, and the density follow immediately (we reinsert H_0 for convenience):

$$\frac{H^2}{H_0^2} = (1+z)^3. \tag{27}$$

$$\frac{a(t)}{a(t_0)} = \left(\frac{t}{t_0}\right)^{2/3}\ ;$$

$$1+z = \left(\frac{t_0}{t}\right)^{2/3} = \left(\frac{2}{3H_0\,t}\right)^{2/3}\ ; \tag{28}$$

$$\frac{\rho_b}{\rho_{crit}} = \frac{H^2}{H_0^2}\ \Omega_{b0} = (1+z)^3\ \Omega_{b0}\ . \tag{29}$$

The age of the Universe can be got from (28), by putting $z = 0$. One obtains $t_0 = 2/(3H_0) \approx 6.5 \times 10^9$ years, a rather small number. There is, as said, a serious flaw in this exercise-model: it supposes that matter dominates down to $t \approx 0$, which is is far from being the case. Furthermore, the protons cannot,

of course, be non-relativistic at the high temperatures of the "beginning" and matter pressure should be added. Let us see two more realistic cases.

b) Matter–domination: present time

Let us go back to the general solution (26) and fix the integration constant by the present value

$$\frac{1}{H\,(t_0)} = \frac{1}{H_0} = \frac{3}{2}\,t_0 + C\ .$$

The solution, now more realistic, will be

$$H(t) = \frac{H_0}{1 + \frac{3}{2}H_0(t - t_0)}. \tag{30}$$

Integrations of $\frac{da}{a} = H(t)dt$ leads then to

$$1 + z = \frac{a_0}{a(t)} = \frac{1}{[1 + \frac{3}{2}H_0(t - t_0)]^{2/3}}. \tag{31}$$

Equation (29) keeps holding. Using that equality, and the value (9) of the critical density, we find

$$\rho_b = 1.878 \times 10^{-26}\ (1 + z)^3\ \Omega_{b0}\ h^2\ [kg\ m^{-3}] \tag{32}$$

Dividing by the proton mass, the number density is

$$n_b = 11.2 \times\ (1 + z)^3\ \Omega_{b0}\ h^2\ \ [m^{-3}] \tag{33}$$

Actually, $\Omega_{b0} = 1$ in the reference case we are considering. This gives a few nucleons per cubic meter at present time. The age of the Universe is basically the same as that for the dust Universe: we look for the time t corresponding to $z \to \infty$, and find $t_0 - t = 2/(3H_0)$. Equations (30) and (31) hold from the turning point down to present times (provided $k = 0$ and $\Lambda = 0$).

c) Matter–domination: k = 0 but $\Lambda \neq 0$

Recent evidence for $k = 0$ and a nonvanishing cosmological constant at present time gives to this case the prominent role.

Let us insert (8) into (29), to get

$$\rho = \rho_0\ e^{-\ 3\int_{t_0}^{t} H(t)dt}$$

and then insert this expression into the Friedmann equation (13):

$$H^2 = 2\left(\frac{4\pi G}{3}\right)\rho_0 \; e^{-\;3\int_{t_0}^{t} H(t)dt} + \frac{\Lambda c^2}{3}\;.$$

The time derivative gives

$$\frac{dH}{dt} = \frac{3}{2}\left(\frac{\Lambda c^2}{3} - H^2\right).$$

Integration leads then to the expression

$$H(t) = \sqrt{\frac{\Lambda c^2}{3}}\;\frac{\left(\sqrt{\frac{\Lambda c^2}{3}} + H_0\right)e^{3\sqrt{\frac{\Lambda c^2}{3}}(t-t_0)} - \left(\sqrt{\frac{\Lambda c^2}{3}} - H_0\right)}{\left(\sqrt{\frac{\Lambda c^2}{3}} + H_0\right)e^{3\sqrt{\frac{\Lambda c^2}{3}}(t-t_0)} + \left(\sqrt{\frac{\Lambda c^2}{3}} - H_0\right)}\;. \tag{34}$$

This expression for $H(t)$ gives $H = H_0$ when $t \to t_0$ and tends to (30) when $\Lambda \to 0$. To have it in terms of more accessible parameters, we may rewrite it as

$$H(t) = H_0\sqrt{\Omega_\Lambda}\;\frac{\left(\sqrt{\Omega_\Lambda}+1\right)e^{3H_0\sqrt{\Omega_\Lambda}(t-t_0)} - \left(\sqrt{\Omega_\Lambda}-1\right)}{\left(\sqrt{\Omega_\Lambda}+1\right)e^{3H_0\sqrt{\Omega_\Lambda}(t-t_0)} + \left(\sqrt{\Omega_\Lambda}-1\right)}\;\;. \tag{35}$$

To get the relation with z, we notice that

$$H^2 = 2\left(\frac{4\pi G}{3}\right)\rho_0(1+z)^3 + \frac{\Lambda c^2}{3} = H_0^2\left[\Omega_b(1+z)^3 + \Omega_\Lambda\right]$$

gives

$$H^2 - \frac{\Lambda c^2}{3} = 2\left(\frac{4\pi G}{3}\right)\rho_0(1+z)^3$$

$$H_0^2 - \frac{\Lambda c^2}{3} = 2\left(\frac{4\pi G}{3}\right)\rho_0\;,$$

which together imply

$$\frac{H^2 - \frac{\Lambda c^2}{3}}{H_0^2 - \frac{\Lambda c^2}{3}} = (1+z)^3\;.$$

Alternatively,

$$(1+z)^3 = \frac{H^2 - H_0^2\;\Omega_\Lambda}{H_0^2\;(1-\Omega_\Lambda)}\;; \tag{36}$$

$$\Omega_b + \Omega_\Lambda = 1\;. \tag{37}$$

It remains to use (34) to obtain

$$1+z=\left(4\ \frac{\Lambda c^2}{3}\right)^{1/3}\frac{e^{\sqrt{\frac{\Lambda c^2}{3}}(t-t_0)}}{\left[\left(\sqrt{\frac{\Lambda c^2}{3}}+H_0\right)\ e^{3\sqrt{\frac{\Lambda c^2}{3}}(t-t_0)}+\left(\sqrt{\frac{\Lambda c^2}{3}}-H_0\right)\right]^{2/3}}\ , \quad (38)$$

which is the same as

$$\frac{a_0}{a(t)}=1+z=(4\ \Omega_\Lambda)^{1/3}\frac{e^{H_0\sqrt{\Omega_\Lambda}(t-t_0)}}{\left[\left(\sqrt{\Omega_\Lambda}+1\right)\ e^{3H_0\sqrt{\Omega_\Lambda}(t-t_0)}+\left(\sqrt{\Omega_\Lambda}-1\right)\right]^{2/3}}\ . \quad (39)$$

The neglected matter pressure is given by

$$\frac{p_b}{c^2\rho_{crit}}=9.2\times 10^{-14}\ T_b\ \ (1+z)^3\ \ \Omega_{b0}\ . \quad (40)$$

This will be one of our main points. Matter pressure is neglected in usual treatments for the reasons given above, which assume an ideal gas. We intend to take into account interactions between nucleons, and shall find indications of a very abrupt raise of this pressure during the PNS period.

2.2 Radiation–dominated age

For photons, the temperature behaves as a frequency so that, by the very definition of red–shift, $T_\gamma = T_{\gamma 0}\ (1{+}z)$. For example, hydrogen recombination takes place at $T_\gamma = T_b \approx 3000K$. This, together with the present value $T_{\gamma 0} \approx 2.7$ for the thermal background, gives a red–shift $(1+z) \approx 1.1\times 10^3$. The mass–equivalent density is therefore

$$\frac{\rho_\gamma}{\rho_{crit}}=2.1\times 10^{-7}\ T_\gamma^4\ h^{-2}=1.1\times 10^{-5}\ (1+z)^4\ h^{-2}=\Omega_{\gamma 0}\ (1+z)^4\ , \quad (41)$$

where

$$\Omega_{\gamma 0}=1.1\times 10^{-5}\ h^{-2}\ . \quad (42)$$

Consider again the reference case $k=0$, $\Lambda=0$. Because $\epsilon_\gamma = 3\ p_\gamma$ and $\rho_\gamma = \frac{\epsilon_\gamma}{c^2}$, we have

$$\dot{H}=-\ 2\ H^2=-\ 2\ \frac{\rho_\gamma}{\rho_{crit}}\ ,$$

leading automatically to

$$H^2 = \Omega_{\gamma 0}(1+z)^4 \ . \tag{43}$$

Solving the equation is only necessary to fix the relation between the time parameter and the red–shift. The solution,

$$H(t) = \frac{1}{2t} \ ,$$

implies

$$t = \frac{1}{2\sqrt{\Omega_{\gamma 0}}(1+z)^2} \quad ; \quad \frac{a(t)}{a_0} = \Omega_{\gamma 0}^{1/4}\sqrt{2t} \ . \tag{44}$$

Before recombination (that is, for higher z's) there is thermal equilibrium between matter and radiation, because electrons are free and the mean free path of the photons is very small. An estimate of the energy per photon at a certain z can be obtained from $kT_{\gamma 0} = 2.3 \times 10^{-10} MeV$, which leads to

$$kT_\gamma = kT_{\gamma 0}(1+z) = 2.32 \times 10^{-10}(1+z) \ MeV. \tag{45}$$

For example, an energy of 4 MeV corresponds to $z \approx 2 \times 10^{10}$. Thus, the thermalized state before recombination makes of T_γ, or its corresponding red–shift, the best time parameter. We shall retain for later use the expressions

$$\frac{p_\gamma}{c^2 \rho_{crit}} = \frac{1}{3} \ \frac{\rho_\gamma}{\rho_{crit}} = 7. \times 10^{-8} \ T_\gamma^4 \ h^{-2} = \frac{\Omega_{\gamma 0}}{3} \ (1+z)^4 \tag{46}$$

$$\frac{\rho_\gamma}{\rho_b} = 2.31 \times 10^{-5} \ (1+z) \ \Omega_{b0}^{-1} \ . \tag{47}$$

At recombination time, $\frac{\rho_b}{\rho_\gamma} \simeq 39.5 \ \Omega_{b0}$. The scale parameter, and consequently the red–shift, behave quite differently in a matter–dominated Universe (28) and in a radiation–dominated one (44). Equation (47) shows that radiation becomes dominant at high z's. The turning point, or change of regime, takes place when $\rho_\gamma \simeq \rho_b$, or

$$1 + z \simeq 4.3 \times 10^4 \ \Omega_{b0} \ . \tag{48}$$

This corresponds to $t \simeq 7.4 \times 10^{-8} \ \Omega_{b0}^{-3/2}/H_0 = 2.2 \times 10^{10} \ \Omega_{b0}^{-3/2}$ sec. When there is no thermalization, non-interacting matter pressure is negligible with respect to radiation pressure by a factor $\frac{p_b}{p_\gamma} \approx \frac{10^{-10}}{1+z} \ T$.

With our proton-related variables, the temperature during the thermalized period preceding recombination will be

$$\tau = \frac{kT_\gamma}{mc^2} = 2.5 \times 10^{-13}(1+z) \ . \tag{49}$$

In terms of those variables we shall have, for example,

$$n_\gamma = 3.8 \times 10^{-39} \ (1+z)^3 \lambda_p^{-3} = 0.24 \left(\frac{\tau}{\lambda_p}\right)^3 \tag{50}$$

and

$$n_b = 6.3 \times 10^{-9} \ \Omega_{b0} h^2 \ \left(\frac{\tau}{\lambda_p}\right)^3 \ . \tag{51}$$

2.3 The Friedmann solutions

Let us now go back to the general equations (19) and (20). Thermalization at $z > 10^3$ has important formal consequences. Dependence of a single temperature makes it more convenient to use the variable z, and much of the discussion can be made in terms of energies, by using (45). Using the units given in Eq.(18), adding matter (29) and radiation (41) densities and extracting from (17) the value of Ω_{b0}, Eq. (19) becomes

$$H^2(z) = \Omega_{\gamma 0}(1+z)^4 - \frac{k}{a_0^2} \ (1+z)^2 + \frac{\Lambda}{3} + (1+z)^3 (1 + \frac{k}{a_0^2} - \frac{\Lambda}{3} - \Omega_{\gamma 0}) \ . \tag{52}$$

This will be modified when interactions between nucleons are taken into account (see equation (64) below). In the reference case,

$$H^2 = \Omega_{\gamma 0}(1+z)^4 + (1 - \Omega_{\gamma 0})(1+z)^3 \ , \tag{53}$$

which reduces to (27) when the last term dominates the right-hand side, and to (43) when the first term dominates.

All this can be see from another point of view. In fact, we have $(1+z)H(t) = -\frac{dz}{dt}$. We can consequently introduce the function $f(z) = \frac{H^2(z)}{H_0^2}$, with $f(0) = 1$ and $a(z) = \frac{a_0}{1+z}$. In the units (18), equation (20) becomes

$$(1+z) \ \frac{df}{dz} = 3 \ f + 3 \ \frac{p}{c^2 \rho_{crit}} + \frac{k}{a_0^2} \ (1+z)^2 - \Lambda \ . \tag{54}$$

Use now x = 1+z, with $f(x = 1) = 1$. As radiation pressure largely dominates at all time, the only contribution to p will come from (46). We shall take $3\,\frac{p}{c^2\rho_{crit}} = \Omega_{\gamma 0}x^4$, with $\Omega_{\gamma 0}$ as given in (42), and rewrite the equation as

$$x\,\frac{df}{dx} = 3\,f + \Omega_{\gamma 0}x^4 + \frac{k}{a_0^2}\,x^2 - \Lambda. \tag{55}$$

The solution is

$$f\,(x) = \Omega_{\gamma 0}x^4 - \frac{k}{a_0^2}\,x^2 + \frac{\Lambda}{3} + x^3(1 + \frac{k}{a_0^2} - \frac{\Lambda}{3} - \Omega_{\gamma 0})\,, \tag{56}$$

which is the same as (52). This point of view will be of interest when the pressure of matter is added. This is due to the fact that pressure appears only in the equation (14) for the derivative of H and not in the expression (13) for H^2.

Things are not that simple for the expansion parameter, or for the relation between z and t. Noting that $x = a_0/a$, we have

$$a\,\frac{da}{dt} = \sqrt{\Omega_{\gamma 0}a_0^4 - ka^2 + \frac{\Lambda}{3}\,a^4 + a_0^3(1 - \Omega_{\gamma 0} + k - \frac{\Lambda}{3})\,a},$$

whose solution is given by

$$t = t_0 + \int_{a(t_0)}^{a(t)} \frac{y\,dy}{\sqrt{\Omega_{\gamma 0}a_0^4 - ky^2 + \Lambda y^4/3 + y\left(1 - \Omega_{\gamma 0} + ka_0^{-2} - \Lambda/3\right)a_0^3}}\ . \tag{57}$$

Most aspects for $z > 10^3$ can be discussed in terms of the red–shift, and we shall have little use for the time variable.

3 Microphysics at the PNS period

There are many interesting questions concerning the applicability of usual physical assumptions and consequent results in the period between $z \approx 10^{15}$ ($kT \approx 400\ GeV$) and $z \approx 10^{10}$ ($kT \approx 4\ MeV$). They are worth a brief parenthesis, as they constitute the background to the discussion of the particular questions we shall be concerned with.

First, concerning Particle Physics, it may be that the usual treatment of cross–sections need revision, due to scarcity of space. In that treatment,

particles are supposed to start in an interaction–free asymptotic region, interact in some finite intermediate region, and finish the process as a free object in yet another asymptotic domain. The ingoing and outgoing flows are compared to define the cross-section. Total volume is supposed to be much larger than the volumes of the particles concerned. Even if for decay rates and production processes with two initial particles (such as pair production) the volume factor cancels out [8], such processes can be inhibited by the smallness of complete (configuration plus momentum) phase space and by final state interactions. The usual elementary definition of the scattering matrix [9] involves a Dirac delta function in four momentum which actually assumes a very large volume. This is a rather difficult question, but one which can in principle be theoretically solved.

Other, deeper Particle Physics aspects are to be considered. Nucleons are non-relativistic at $z \approx 10^{10}$, but highly relativistic at $z \approx 10^{15}$. How far will a proton, when energy grows from 4 MeV to 400 GeV, remain a proton ? Is it true that at the energies involved in the period we can already talk of deconfined quarks and unshielded gluons ? It is usually supposed that at high enough energies nucleons loose their identities and the system must be considered as a quark–gluon plasma [10]. The energy at which that happens, however, remains unknown. The search for a signal of quark deconfinement, which has been actively looked for in the interval $15-200$ GeV per proton, has not yet given a definitive answer [11]. The experimental results are consistent with the presence of deconfinement, but do not exclude other interpretations. This means that the signal is not unambiguous, is not conclusive [12]. What happens at still higher energies is simply not known, as the mechanisms of confinement and eventual deconfinement are as yet unclear even from the purely theoretical point of view [13]. We shall here suppose that protons keep their identities and examine the consequences. We shall find, as we proceed backwards in time, that radiation protons produce a pressure blockage at energies of the order of hundreds of MeV, much too low for confinement to take place.

Concerning Thermodynamics and Statistical Mechanics aspects, the system should be treated, quite probably, as a finite system [14]. Another point concerns fermions in general. The Pauli principle can be seen as a consequence of an effective repulsive potential between kin fermions, whose range is the thermal wavelength. This wavelength decreases with temperature. At the period under consideration, it will be very small for electrons, but large for non-relativistic protons. What has been said of phase space for cross

sections should be repeated in this context: pair production of protons, for example, would be inhibited by the presence of many protons in the final state.

Anyhow, the first problem to be faced is much simpler. The picture we have of the early Universe comes from inserting *ideal* fluid equations of state in the Friedmann equations. In the PNS period, excepting the reactions leading to nucleosynthesis and their nuclei–breaking inverses, interactions are not taken into account at all, let alone the possibility of any abrupt behavior related to phase-transitions. Thus, the first thing to be done would be to consider a real, interacting gas. This is already a difficult enough task. We shall make a first attempt by taking into account the size of the nucleons. Protons and neutrons are, of course, the hardest known objects. Instead of structureless, pointlike particles, we shall assume a hard–sphere gas.

A negative point is that the only known means to do it is by considering a hard-sphere potential, and potentials loose progressively their meaning when the particles represented become more and more relativistic. Taking into account pair creation and annihilation become more and more necessary to avoid difficulties akin to the Klein paradox [9]. We shall do it as carefully as possible, in the hope of finding effects in the non-relativistic regime. The nucleon size–effect will be simulated by a hard-sphere with the nucleon radius. What we should take for the proton radius is as yet a matter of controversy [15], but the uncertainty between 0.80 fermi and 0.86 fermi is, of course, irrelevant for the gross estimate we have in view. We shall use 0.8×10^{-13} cm for the proton radius and undertake the usual backward path, from $z \approx 10^{10}$ to $z \approx 10^{15}$.

4 A solvable model for primeval close packing

We have presented in the introduction a very rough estimate of the red–shift at which a blockage can take place, and proceed now to a somewhat more elaborate evaluation. The result will be essentially the same as long as only the remnant protons are concerned. This more refined approach provides, however, general formulae which can be applied also to the protons belonging to the radiation.

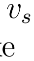
The hard-sphere gas is one of the great unsolved problems of Theoretical Physics. The virial coefficients have been calculated analytically only up to the fourth order (by Boltzmann). Numerical results exist for higher orders,

but the virial series has, if any, a poor convergence. The best picture of the system is given by a computer simulation refined through the use of Padé approximants. The outcome is a curve for the equation of state [16]. It shows a clear phase transition (possibly two), in which the pressure grows steeply to ∞. The curve can be parametrized by an equation of state of the type

$$p_b = \frac{n_b\, kT}{1 - n_b/n_c}, \tag{58}$$

where $n_c = \frac{\sqrt{2}}{D^3}$, D being the sphere diameter. This equation can be qualitatively understood in the "excluded volume" approach to the hard sphere gas. The canonical partition function for an N-particle gas with interactions given by a potential $V_{ij} = V(|\mathbf{r}_i - \mathbf{r}_i|)$ is

$$\begin{aligned} Q_N &= \frac{1}{\lambda^{3N}} \int d^3r_1 d^3r_2 \ldots d^3r_N \; e^{-\beta \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} V_{ij}} \\ &= \frac{1}{\lambda^{3N}} \int d^3r_1 d^3r_2 \ldots d^3r_N \prod_{i=1}^{N-1} \prod_{j=i+1}^{N} e^{-\beta V_{ij}}. \end{aligned}$$

For a hard–sphere potential, the integrand vanishes every time it happens that $|\mathbf{r}_i - \mathbf{r}_j| < D/2$ for any pair (i, j) of particles. Thus, it all amounts to forbid the regions with $|\mathbf{r}_i - \mathbf{r}_j| < D/2$ for every pair, or to exclude the interior of all the spheres. As the potential simply vanishes outside the spheres, the final picture is that of an ideal gas in a volume reduced by the total volume of the N spheres. With the volume $v_s = \frac{\pi}{6}\, D^3$ for each sphere, the equation would be $p(V - Nv_s) = NkT$, or $p = nkT/(1 - nv_s)$. There are actually some geometric factors before v_s, as the excluded volume increases at close packing. In a configuration like  each sphere actually excludes a cube of size D. These geometric factors have been found by the authors of [16], the second suggested phase transition corresponding to the still tighter packing of type .

Let us now apply (58) to our problem. The proton radius gives $D^3 = 4.1 \times 10^{-39}\ cm^3$, so that $n_c = 0.345 \times 10^{39}\ cm^{-3}$. Using (10), we find

$$\frac{n_b}{n_c} = 3.18 \times 10^{-44}\ (1 + z)^3\ \Omega_{b0}\ h^2\ . \tag{59}$$

The denominator in (58) vanishes when $1 + z = \frac{3.1 \times 10^{14}}{\Omega_{b0}^{1/3}\ h^{2/3}}$, corresponding to

$$kT \approx \frac{73.6}{\Omega_{b0}^{1/3}\ h^{2/3}}\ GeV\ .$$

Thus, if $h = 0.7$ and $\Omega_{b0} = 0.03$, then $kT \approx 300\ GeV$. The recently more favored value $\Omega_{b0} = 0.2$ would give $kT \approx 160\ GeV$. This is an indication of close-packing for the protons existing today, but at the very high value $z \approx 10^{15}$. The close-packing effect will actually take place at much lower z because of the pair–production effect, to be discussed in the next section. Notice anyhow that, even neglecting the pair–produced protons, the proton mean free path would be $\approx (n_c \lambda_P^2)^{-1}$, comparable to λ_P itself.

Using (40), the baryon pressure (58) will be given by

$$\frac{p_b}{c^2 \rho_{crit}} = 9.2 \times 10^{-14}\,(1+z)^3\ \ \Omega_{b0}\ T\ \frac{1}{1 - 3.18 \times 10^{-44}\ (1+z)^3\ \ \Omega_{b0}\ h^2}\ . \quad (60)$$

As there is thermal equilibrium at the period of interest, T will be the radiation temperature and consequently

$$\frac{p_b}{c^2 \rho_{crit}} = 2.48 \times 10^{-13}\ \Omega_{b0}\ \ (1+z)^4\ \ \frac{1}{1 - 3.18 \times 10^{-44}\ (1+z)^3\ \ \Omega_{b0}\ h^2}\ .$$

The relation between this and the radiation pressure is

$$\frac{p_b}{p_\gamma} = 3.17 \times 10^{-8}\ \Omega_{b0}\ h^2\ \ \frac{1}{1 - 3.18 \times 10^{-44}\ (1+z)^3\ \ \Omega_{b0}\ h^2}\ .$$

We can introduce the notation

$$M = 2.48 \times 10^{-13}\ \Omega_{b0}\ h^2, \quad Q = 3.15 \times 10^{-15}\ \ \Omega_{b0}^{1/3}\ h^{2/3} \quad (61)$$

and write

$$\frac{p_b}{c^2 \rho_{crit}} = M\,(1+z)^4\ \frac{1}{1 - [Q\ (1+z)]^3}\ . \quad (62)$$

This is to be added to the Friedmann equation in the form (55), which becomes

$$x\ \frac{df}{dx} = 3\ f + \Omega_{\gamma 0} x^4 + \frac{k}{a_0^2}\ x^2 + M x^4 \frac{1}{1 - (Qx)^3} - \Lambda\ . \quad (63)$$

The solution is

$$f(x) = \Omega_{\gamma 0} x^4 - \frac{k}{a_0^2} x^2 + \frac{\Lambda}{3} + x^3 \left[1 + \frac{k}{a_0^2} - \frac{\Lambda}{3} - \Omega_{\gamma 0}\right] + \frac{M}{3} \times$$

$$x^3 \left\{ \ln \frac{(1-Q)\sqrt{1 + Qx + Q^2 x^2}}{(1-Qx)\sqrt{1+Q+Q^2}} + \arctan \frac{\sqrt{3}\ Qx}{2+Qx} - \arctan \frac{\sqrt{3}\ Q}{2+Q} \right\}. \quad (64)$$

This is just (56), with the additional term proportional to M. As $Q \ll 1$, a good approximation is

$$f(x) = \Omega_{\gamma 0} x^4 - \frac{k}{a_0^2}\, x^2 + \frac{\Lambda}{3} + x^3 \left[1 + \frac{k}{a_0^2} - \frac{\Lambda}{3}\right]$$
$$+ \frac{M}{3}\, x^3 \left\{ \ln \frac{\sqrt{1 + Qx + Q^2 x^2}}{1 - Qx} + \arctan \frac{\sqrt{3}\; Qx}{2 + Qx} \right\}. \quad (65)$$

When x tends to Q^{-1} from smaller values, the functions $f(x)$ and $H(z)$ become infinite. The implicit solution for $a(t)$ is still given by (57), with $f(x)$ now given by (64).

We arrive thus at the following provisional picture. If we proceed backwardly in time, there will be a value of the increasing red-shift for which matter pressure produced by the remnant protons becomes practically infinite. This value corresponds to energies of a few hundreds of GeV. Of course, at such energies the very notion of potential is unacceptable. But this is only a first step in our reasoning chain. Actually, there will be a much larger number of protons in the medium. We have up to now neglected those which are produced in pairs by the radiation background. If that number is high enough, the pressure may become exceedingly high at lower energies, in which potentials still do make sense.

5 Radiation-belonging nucleons

The radiation background contains a large amount of massive particles as soon as the pair production $\gamma\gamma \to e^- \; e^+$ threshold is attained. As to the process of hadron production $\gamma\gamma \to p\bar{p}$, it is clearly at work at kT around $1\ GeV$. Actually, the reaction threshold is much lower [17], because of the huge number of photons. Even at lower energies, there are many photons with energy high enough to produce pairs. The number of radiation-created hadrons becomes of the same order of the number of photons a little above the threshold [18], much larger in effect than the number of remnant protons considered previously. Let us say a few more words on these statements.

The basic question would be: when we proceed backwards in time, from which energy on can we consider that the reaction $\gamma\gamma \to p\bar{p}$ is in equilibrium ? This energy is important because, above it, the remnant protons

are negligible and the previous argument should be applied, instead, to the radiation-belonging protons.

With an annihilation cross section $\sigma_a \approx 2 \times 10^{-26} cm^2$, the annihilation mean free path of a proton is $\lambda_a \approx \frac{1}{n_\gamma \sigma_a} \approx 10^{23}(1+z)^{-3}$ cm. This gives $\approx 10^{-22}$ cm at 400 GeV, $\approx 2 \times 10^{-21} cm$ at $20 GeV$, and 10^{-7} cm at 4 MeV. For the inverse reaction, pair creation, we could use the Breit–Wheeler cross-section [19], but the exact value is not necessary in our simplified approach. An estimate, using only mean free paths, can be made along the following lines. The mean free path for a photon, due to pair creation, is $\lambda_{\gamma\gamma\to p\bar{p}} = [n_\gamma \sigma_{\gamma\gamma\to p\bar{p}}]^{-1}$. This means that, on the average, a γ will meet another γ to produce a pair every time the volume $\lambda_{\gamma\gamma\to p\bar{p}}\ \sigma_{\gamma\gamma\to p\bar{p}}$ is spanned by a γ. Thus, a traveling photon will "deposit" one antiproton at each volume of that value. But that volume is just $1/n_\gamma$, so that the concentration of antiprotons is roughly the same as that of the photons.

A more precise description of the interplay between annihilation and pair production requires a detailed analysis of the kinetics involved. A kinetic estimate [18] gives $\tau \approx 1/44 \approx 0.02$ (corresponding to $\approx 20\ MeV$) for the temperature above which there is equilibrium. We insist that, as soon as chemical equilibrium is attained, at kT $\approx 20\ MeV$, the number of antiprotons becomes enormous, of the order of n_γ.

We can also estimate the temperature at which the number of pair–produced protons (or antiprotons) becomes larger than the number of remnant protons, by equating (85) and (51). The result of a numerical analysis is that the number of antiprotons overcomes that of the remnant protons at $\tau \approx 0.058$. Thus, we can choose for security a reasonable value ≈ 0.06 (corresponding to $\approx 60\ MeV$) and say that, for τ above it, the reaction is in equilibrium and there will be a large amount of protons and antiprotons. At $kT \approx 60\ MeV$, these protons and antiprotons are nonrelativistic and for them the potential–based arguments are valid.

We have repeatedly said that the concentration of protons is actually of the same order of magnitude of that of photons, many orders of magnitude above the number of the remnant protons we have considered in the previous section. In fact, using the numerical factor $n_\gamma/n_{\bar{p}} = 4/3$ discussed at the end of Appendix A, and equations (50) and (51), we find

$$\frac{n_{\bar{p}}}{n_b} = 2.9 \times 10^7\ [\Omega_b h^2]^{-1}\ . \tag{66}$$

There is consequently an abrupt jump in the concentration, and (59) will

change dramatically. To take this effect into account, it is sufficient to change the numerical parameters (61): M and Q^3 must be multiplied by $\frac{n_{\bar{p}}}{n_b}$, given by (66). Thus, the formulae of the previous section can be used, with the parameters M and Q of (61) replaced by

$$M' = \frac{n_{\bar{p}}}{n_b}\ M = 7.2 \times 10^{-6}\ ;\ Q' = \left(\frac{n_{\bar{p}}}{n_b}\right)^{1/3}\ Q = 9.56 \times 10^{-13}\ . \tag{67}$$

In that case, $\frac{p'_b}{c^2 \rho_{crit}} = M'\,(1+z)^4 \, \frac{1}{1-[Q'\ (1+z)]^3}$, or, as $M' = 0.65 h^2 \Omega_{\gamma 0}$,

$$\frac{p'_b}{c^2 \rho_{crit}} = \frac{1.95 h^2}{1-[Q'\ (1+z)]^3} \frac{p_\gamma}{c^2 \rho_{crit}} = \frac{0.65 h^2}{1-[Q'\ (1+z)]^3}\ \frac{\epsilon_\gamma}{c^2 \rho_{crit}}. \tag{68}$$

The term $\Omega_{\gamma 0} x^4$ in (63) is the radiation contribution. That equation becomes

$$(1+z)\ \frac{df}{d(1+z)} = 3\ f + \frac{k}{a_0^2}\ (1+z)^2 - \Lambda + \Omega_{\gamma 0}(1+z)^4 \left[1 + \frac{0.65\ h^2}{1-[Q'\ (1+z)]^3}\right]. \tag{69}$$

The last term corresponds to the radiation contribution, with the term in h^2 giving the nucleon interaction correction to the equation of state (46). The solution is still (64), but with the replacements $M \to M'$ and $Q \to Q'$.

Matter pressure now becomes infinite at $z = 1.0 \times 10^{12}$, or $kT = 232 MeV$. At such energies, it is possible that deconfinement, conjectured to happen at $\approx 150 MeV$, has taken place. We are, however, neglecting some effects (discussed below) which would tend to lower that value. And even at $\approx 230 MeV$ the non–relativistic argument, based on the notion of a potential, is valid. Notice that the energy at which pair production attains equilibrium is much lower: $\tau \approx 0.05$ corresponds to $kT_\gamma \approx 50\ \ MeV$. A last point is that antiprotons attain a concentration comparable to that of photons a bit above the reaction threshold, which can be lower than the equilibrium temperature. We are, thus, quite probably overestimating the above energies.

6 Conclusions and speculations

The conclusion is that, as soon as pair creation starts up, the number of protons becomes so high as to create a blockage. This might mean that the reaction is inhibited by a “wall” of occupied phase space in the final state, or that the use of cross–sections is inadequate, or still that usual thermodynamics does not apply. It is common knowledge that, in consequence of

fundamental requirements, pressure in a fluid cannot go to infinity. It can at most attain the incompressibility limit, at which the equation of state is $p = \epsilon$ and beyond which causality would be violated. With $h = 1$, this point would correspond to $z \approx 10^{12}$.

In its details, the primeval blockage we have dealt on has, unfortunately, a crucial dependence on the values of the as yet undetermined cosmological parameters. Some of the neglected aspects would, however, add to the repulsion and, consequently, to the effect. Notice that any effect leading to a higher effective value for the sphere volumes in (58) will bring the critical redshift to lower values. For instance, scattering theory will tell us that a sphere appears larger in the quantum case: naïve shadow scattering by a disk of radius r will see a transverse area $4\pi r^2$, instead of the classical πr^2. This would mean an effective radius twice as large, and a red-shift half that found above ($z = 0.5 \times 10^{12}$, or $kT = 116\ MeV$). Still another possiblility is the breaking of Debye electromagnetic screening, which would lead to an increase of the electrostatic Coulomb repulsion. Finally, we have completely neglected the Fermi repulsion due to Pauli exclusion. This effect is feeble in the ultrarelativistic regime, but the thermal wavelengths (23) or (24) can be larger than the diameter of the excluded sphere for the range of energies found.

We have not attempted to relate our results to the current negative–pressure approach to inflation [20, 21]. We shall only indulge in a wild remark. As Lee and Yang have taught us long ago [22], equations of state have the same analytical form both sides of a phase transition. If equation (58) holds for $n_b/n_c > 1$, the pressure will be negative "the other side" of the thermodynamic singularity. In effect, the expression for the total radiation pressure, including the nucleon–antinucleon pairs,

$$\frac{p_\gamma}{c^2 \rho_{crit}} = \Omega_{\gamma 0}(1+z)^4 \left[\frac{1}{3} + \frac{0.65\ h^2}{1 - [Q'\ (1+z)]^3} \right] , \tag{70}$$

or (using ϵ_γ for the *ideal* energy density)

$$p_\gamma = \left[\frac{1}{3} + \frac{0.65\ h^2}{1 - [Q'\ (1+z)]^3} \right] \epsilon_\gamma = \left[\frac{1}{3} + \frac{0.65\ h^2}{1 - \left[\frac{kT_\gamma (MeV)}{232} \right]^3} \right] \epsilon_\gamma , \tag{71}$$

will be negative in a short interval before the singular red–shift. A comparison of (19) and (52) shows that, for the large values of z we are considering now,

the energy density is dominated by the quartic term. Consequently, the ideal ϵ_γ of the last expressions is the true radiation energy and (71) is indeed an equation of state. The equation would be of the exponential inflationary type $p = -\epsilon$ type [23] only for a very particular value of z. We can however, speculate on the possibility of a more general type of inflation. For a barotropic equation $p = (\gamma - 1)\epsilon$ an extended, power–law type inflation [25] occurs for γ in the range $0 \leq \gamma \leq 2/3$ [24]. For $h = 1$, this would correspond to a tiny interval $1.14 \leq Q' \, (1 + z) \leq 1.25$ before the pressure singularity.

Acknowledgements

We are deeply grateful to Professor R.A. Salmeron, of the École Polytéchnique, Palaiseau, and CERN, Geneva, for illuminating discussions on the experimental evidence concerning quark deconfinement. One of the authors (R.A.) is grateful to FAPESP (São Paulo, Brazil) for financial support.

A Relativistic Gases

We justify here some statements and formulae of the text, and rewrite some of the most usual expressions in units specially adequate to our case.

To see how (23) comes out, let us recall that the grand-canonical partition function for a gas of non-interacting quantum particles with chemical potential μ is the trace of the density operator:

$$\Xi(V, \beta, \mu) = tr \left[e^{-\beta \sum_i (\epsilon_i - \mu)\hat{n}_i} \right] = \sum_{\{n_j\}} \langle n_0 n_1 n_2 \ldots | e^{-\beta \sum_i (\epsilon_i - \mu)\hat{n}_i} | n_0 n_1 n_2 \ldots \rangle =$$

$$\sum_{n_0} \sum_{n_1} \sum_{n_2} \ldots e^{-\beta(\epsilon_0 - \mu)n_0} e^{-\beta(\epsilon_1 - \mu)n_1} e^{-\beta(\epsilon_2 - \mu)n_2} \ldots = \prod_i \sum_n e^{-\beta(\epsilon_i - \mu)n}$$

$$= \prod_\epsilon \sum_n e^{-\beta(\epsilon - \mu)n}.$$

In this non-interacting case, each level contributes an independent factor. The system can have also internal degrees of freedom, which will likewise contribute separately. Suppose a single degree of freedom (spin, for example) taking g possible values. The partition function will be

$$\Xi(V, \beta, \mu) = \prod_\epsilon \left[\sum_n e^{-\beta(\epsilon - \mu)n} \right]^g . \tag{72}$$

The kind of statistics appears in the summation, which is over the possible values of the occupation number n, from $n = 0$ up to the maximum number of particles allowed in each state: 1 for fermions, ∞ for bosons. To treat bosons and fermions at the same time, we adopt the usual convention: upper signs for bosons, lower signs for fermions. Transforming the product into a summation by using the formal identity $\prod_\epsilon\{\ldots\} = \prod_\epsilon[\exp(\ln\{\ldots\})] = exp\sum_\epsilon \ln\{\ldots\}$, the above expressions lead to

$$\ln \Xi^{B,F}(V,\beta,\mu) = \mp g \sum_\epsilon \ln\left[1 \mp e^{-\beta(\epsilon-\mu)}\right] \ . \tag{73}$$

It is convenient to use the fugacity variable, either the usual non-relativistic fugacity $z = e^{\beta\mu}$ or its relativistic version $Z = e^{\beta\mu_R} = ze^{\beta mc^2}$. If we do not care about zero–energy states, the sum over the energy levels can be replaced by an integral over the momenta through the prescription $\sum_\epsilon \to h^{-3}\int d^3x d^3p$, which leads to

$$\ln \Xi^{B,F}(V,\beta,\mu) = \mp g \frac{4\pi V}{h^3}\int_0^\infty p^2 dp \ln\left[1 \mp Ze^{-\beta(p^2c^2+m^2c^4)^{1/2}}\right] \ .$$

Expanding the logarithm and collecting like terms, the partition function acquires the form

$$\Xi^{B,F}(V,\beta,z) = \exp\left\{\frac{gV}{h^3}\sum_{j=1}^\infty \frac{(\pm 1)^{j-1}}{j} z^j \int d^3p e^{-j\beta[(p^2c^2+m^2c^4)^{1/2}-mc^2]}\right\}. \tag{74}$$

The relativistic thermal wavelength (23) appears now in the form

$$\frac{1}{h^3}\int d^3p \ \ e^{-\beta[(p^2c^2+m^2c^4)^{1/2}-mc^2]} = \frac{1}{\Lambda_T^3(\beta)}, \tag{75}$$

and the final expression for the grand-canonical partition function for a gas of non-interacting quantum particles is

$$\Xi^{B,F}(V,\beta,z) = \exp\left\{gV \sum_{j=1}^\infty \frac{(\pm 1)^{j-1}}{j} z^j \frac{1}{\Lambda_T^3(j\beta)}\right\} \ , \tag{76}$$

or its equivalent

$$\Xi^{B,F}(V,\beta,z) = \exp\left\{g\frac{4\pi V}{h^3c^3}\ \frac{(mc^2)^2}{\beta}\ \sum_{j=1}^\infty \frac{(\pm 1)^{j-1}}{j} Z^j K_2(j\beta mc^2)\right\} \ . \tag{77}$$

Here $K_2(x)$ is the modified Bessel function of second order. Limits can be found by using the properties

$$K_2(x) \approx \sqrt{\frac{\pi}{2x}} e^{-x} \left(1 + \frac{15}{8x} + \ldots\right);$$

$$K_1(x) \approx \sqrt{\frac{\pi}{2x}} e^{-x} \left(1 + \frac{3}{8x} + \ldots\right) \text{ for x >> 1;}$$

$$K_2(x) \approx 2x^{-2} \; ; K_1(x) \approx x^{-1} \text{ for x << 1.}$$

$K_1(\beta mc^2)$ will appear only in the energy expression. The non-relativistic and the ultra–relativistic limits give (24) and (25). The pressure and the particle number follow by standard thermodynamic relations:

$$pV = kT \ln \Xi = gkT \sum_{l=1}^{\infty} \frac{(\pm)^{l-1}}{l} z^l \frac{1}{\Lambda_T^3(l\beta)}, \tag{78}$$

$$\begin{aligned} \bar{N} = \left[z \frac{\partial}{\partial z} \; ln \; \Xi(V, \beta, z) \right]_{V,\beta} &= g \sum_{\epsilon} \; \frac{1}{z^{-1} e^{\beta \epsilon} \mp 1} \\ = \frac{gV}{h^3} \int \frac{d^3 p}{z^{-1} e^{\beta \sqrt{p^2 c^2 + m^2 c^4}} \pm 1} &= gV \sum_{l=1}^{\infty} (\pm)^{l-1} z^l \frac{1}{\Lambda_T^3(l\beta)} \; . \end{aligned} \tag{79}$$

The expressions in terms of integrals or of series are more or less convenient, depending on the application in view. We can extract the density number of particles at energy ϵ, $n_\epsilon = g \; [z^{-1} e^{\beta \epsilon} \mp 1]^{-1}$. The average energy, including the masses, is

$$\begin{aligned} \bar{E} = - \left(\frac{\partial}{\partial \beta} \; \ln \Xi(V, \beta, z) \right)_{Z,V} &= \sum_{\epsilon} n_\epsilon \epsilon \\ = 3pV + \; 4\pi \; g \left(\frac{mc^2}{\lambda_C^3} \right) \left(\frac{kT}{mc^2} \right) &\sum_{l=1}^{\infty} \frac{(\pm)^{l-1}}{l} z^l e^{l\beta mc^2} K_1(l\beta mc^2) \; . \end{aligned} \tag{80}$$

The degree of degeneracy is

$$d = \frac{\bar{N} \Lambda_T^3(\beta)}{V} = g \sum_{l=1}^{\infty} (\pm)^{l-1} z^l \frac{\Lambda_T^3(\beta)}{\Lambda_T^3(l\beta)} = g \sum_{l=1}^{\infty} \frac{(\pm)^{l-1}}{l} z^l \frac{e^{-\beta mc^2} K_2(l\beta mc^2)}{e^{-l\beta mc^2} K_2(\beta mc^2)} \; . \tag{81}$$

We are particularly interested in $n = \frac{\bar{N}}{V}$. For a massless particle, the change of variables $x = pc/kT$ can be used directly to give

$$n = \frac{g}{2\pi^2} \; \frac{\tau^3}{\lambda_C^3} \int_0^\infty \frac{x^2 dx}{z^{-1} e^x \pm 1} \; . \tag{82}$$

For a gas of photons (using g = 2, z = 1), there are many expressions of interest:

$$n_\gamma = \frac{\bar{N}_\gamma}{V} = \frac{2}{h^3}\int \frac{d^3p}{e^{\beta pc}-1} = \frac{1}{\pi^2}\;\frac{\tau^3}{\lambda_C^3}\int_0^\infty \frac{x^2dx}{e^x-1} = 2\sum_{l=1}^\infty \frac{1}{\Lambda_{UR}^3(l\beta)} \tag{83}$$

$$= 2\;\frac{\tau^3}{\pi^2\lambda_C^3}\sum_{l=1}^\infty \frac{1}{l^3} = \frac{2\;\zeta(3)}{\pi^2}\left(\frac{kT_\gamma}{\hbar c}\right)^3 = 0.244\;\frac{\tau^3}{\lambda_C^3}\;. \tag{84}$$

The pressure is found to be $p_\gamma = \frac{\zeta(4)}{\zeta(3)}\;n_\gamma kT_\gamma = \bar{E}_\gamma/3V = \epsilon_\gamma/3$. For a gas of fermions with g = 2 (like protons or antiprotons),

$$n_{\bar{p}} = \frac{\bar{N}_p}{V} = 2\sum_{l=1}^\infty(-)^{l-1}z^l\frac{1}{\Lambda_T^3(l\beta)} = n_{\bar{p}} = \frac{1}{\pi^2}\;\left(\frac{\tau}{\lambda_p}\right)^3\int_0^\infty \frac{x^2dx}{z^{-1}e^{\sqrt{1/\tau^2+x^2}}+1}\;. \tag{85}$$

In the ultrarelativistic regime this becomes

$$n_{\bar{p}} = \frac{1}{\pi^2}\;\frac{\tau^3}{\lambda_C^3}\int_0^\infty \frac{x^2dx}{e^x+1} = \frac{3}{2}\;\frac{\zeta(3)}{\pi^2}\;\frac{\tau^3}{\lambda_C^3} = 0.183\;\frac{\tau^3}{\lambda_C^3}\;. \tag{86}$$

A factor $\frac{n_\gamma}{n_{\bar{p}}} = \frac{4}{3}$ comes from the fermion repulsion effect, encapsulated in the sign in the integrand denominator, opposite to that in (83). It is more difficult to pack fermions than bosons together. This can be seen also from the limits of the degeneracy index (81). In this ultrarelativistic regime, its values are, for photons and antiprotons, respectively, $n_\gamma\;\Lambda_{UR}^3 = 0.244\;\pi = 0.766$ and $n_{\bar{p}UR}\;\Lambda_{UR}^3 = 0.183\;\pi = 0.575$. The relativistic or nonrelativistic character of the protons depend on the above numerical factors. They are possibly irrelevant to rough estimates of the main text, but may come to be important in a more detailed consideration of Fermi repulsion.

References

References

[1] Particle Data Group, *Review of Particle Physics*, European Physical Journal **C3**, 1 (1999).

[2] V. Canuto, *Equation of state at ultrahigh densities*, Annual Review of Astronomy and Astrophysics **12**, 167 (1974); and **13**, 335 (1975).

[3] Leonard Parker and Yi Wang, Phys.Rev. **D42**, 1877 (1990).

[4] S. Weinberg, *Gravitation and Cosmology*, J. Wiley, New York, 1972. .

[5] P. Coles and G. F. R. Ellis, *Is the universe open or closed ?* Cambridge University Press, 1997.

[6] S. Perlmutter *et al*, *Nature* **391**, 51 (1998); A. G. Riess *et al*, *A. J.* **116**, 1009 (1998); S. Perlmutter *et al*, *Ap. J.* **517**, 565 (1999); P. de Bernardis *et al*, *Nature* **404**, 955 (2000); S. Hanany *et al*, *Ap.J. Lett.* **545**, 5 (2000).

[7] R. Aldrovandi and I. Monte Lima, Astrophys.Space Sci. **90**, 179 (1983).

[8] S. Weinberg, *The Quantum Theory of Fields*, Cambridge University Press, 1995, Vol. I, section 3.4.

[9] J.D. Bjorken and S.D. Drell, *Relativistic Quantum Mechanics*, McGraw-Hill, New York, 1964.

[10] M.S. Turner and J.A. Tyson, Rev.Mod.Phys. **71**, S145 (1999).

[11] U. Heinz, *Hunting Down the Quark–Gluon Plasma in Relativistic Heavy-Ion Collisions*, preprint CERN-TH/99-45, hep-ph/9902424, 1999.

[12] R.A. Salmeron, private communication.

[13] M.K. Gaillard, P.D. Grannis and F.J. Sciulli, Rev.Mod.Phys. **71**, S96 (1999).

[14] T. Hill, *Thermodynamics of Small Systems*, Dover, New York, 1994.

[15] T. Kinoshita, in T. Kinoshita (ed.), *Quantum Electrodynamics*, World Scientific, Singapore, 1990.

[16] V. C. Aguilera–Navarro, R. F. T Souza, M. de Llano and S. Mini, Physica **128B**, 190 (1985).

[17] L.D. Landau and E.M. Lifschitz, *Statistical Mechanics*, Pergamon Press, 1970.

[18] Ya. B. Zeldovich and I. D. Novikov, *Relativistic Astrophysics II: The Structure and Evolution of the Universe*, University of Chicago Press, 1981.

[19] V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii, *Relativistic Quantum Theory*, Pergamon Press, 1971.

[20] W.H. McCrea, Proc.Roy.Soc. (London) **206**, 562 (1951).

[21] George L. Murphy, Phys.Rev. **D8**, 4231 (1973).

[22] T.D. Lee and C.N. Yang, Phys. Rev. **87**, 410 (1952) and Phys. Rev. **113**, 1165 (1959).

[23] C. Wolf, Nuovo Cim. **107B**, 987 (1992).

[24] G.F.R. Ellis, Standard and Inflationary Cosmologies, Lectures given at the Summer Research Institute, Banff Centre, Banff, Alberta, Canada, 1990.

[25] F. Lucchin and S. Matarese, Phys. Rev. **D32**, 1316 (1985).